

## **Dynamics and Stability of Acoustic Wavefronts in the Ocean**

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### **LONG-TERM GOALS**

- To develop a method of modeling sound propagation in an environment with multi-scale inhomogeneities, which preserves the efficiency and intuitive qualities of the ray theory but is free from spurious environmental sensitivity and strong perturbations associated with ray trajectories.
- To investigate and quantify effects on underwater acoustic wavefronts of internal gravity waves, sea swell, “spice,” and other small-scale processes in the water column.

### **OBJECTIVES**

1. To assess significance of time dependence of the sound speed and flow velocity perturbations on predictability of acoustic wavefronts and timefronts.
2. To quantify horizontal refraction of sound by random meso-scale inhomogeneities at  $O(1)$ Mm propagation ranges.
3. To find the variance and bias of random ray travel times in the regime, where the ray displacement may be comparable to the vertical extent of the underwater waveguide but the clustering has not developed yet.
4. To determine, using a perturbation theory and numerical simulations, typical propagation ranges where clustering of chaotic rays replaces the anisotropy of ray scattering as the main physical mechanism responsible for acoustic wavefront stability.
5. To develop an efficient technique for modeling acoustic wavefronts and their perturbations in range-dependent and horizontally inhomogeneous oceans.
6. To model perturbations of acoustic wavefronts and timefronts by internal gravity waves, internal tides, sea swell, and “spice” in the ocean.
7. To determine, using a perturbation theory and full-wave numerical simulations, the range of acoustic frequencies where diffusion along the wavefront overtakes the anisotropy of ray scattering as the main physical mechanism responsible for acoustic wavefront stability.

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8. To investigate implications of wavefront stability on the downward extension of acoustic timefronts and deepening of lower turning points of steep rays due to small- and meso-scale physical processes in the upper ocean.

## APPROACH

Our theoretical approach is an extension of the method employed in (Godin, 2003, 2007, 2009), where a novel perturbation technique has been developed to solve the eikonal equation and calculate wavefront and ray trajectory displacements, which are required to be small over a correlation length of the environmental inhomogeneities but not necessarily over the entire acoustic propagation path. In order to extend analysis to longer ranges, this method will be combined with perturbation approaches developed (for 2-D problems) in the context of the ray chaos theory (Brown et al., 2003; Virovlyansky, 2006; Makarov et al., 2010).

The analytical methods will be complemented by numerical wavefront-tracing techniques. A major problem with a direct modeling of acoustic wavefronts in the ocean through numerical solution of the eikonal equation lies in the eikonal (and acoustic travel time) being a multi-valued function of position. A number of computational approaches to solve the eikonal equation without ray tracing have been developed in mathematical and seismological communities (Vidale, 1990; Sava and Fomel, 2001; Sethian, 2001; Benamou, 2003). However, most of these methods are only capable of tracing the wavefront of the earliest, first arrival and thus are not suitable for underwater acoustic applications. We plan to adapt to ocean acoustics problems the grid-based and Lagrangian wavefront construction techniques (Vinje et al., 1993, 1999; Lambaré et al., 1996; Sava and Fomel, 2001; Benamou, 2003; Chambers and Kendall, 2008; Hauser et al., 2008), which have been developed in the context of exploration seismology and are capable of computing all arrivals. Extension of the existing wavefront construction techniques to long-range underwater sound propagation is a non-trivial and, in fact, rather challenging task because the number of ray arrivals and topological complexity of wavefronts in the ocean far exceed those in the geophysical applications considered to date. Following the ideas presented in (Godin, 2002), the wavefront construction techniques will be extended to moving and non-stationary (i.e., time-dependent) media. Reciprocity and wavefront reversibility considerations will play a significant role in developing a computationally efficient approach to numerical modeling of acoustic wavefronts and timefronts in the ocean.

Theoretical results and new modeling capabilities will be verified against numerical simulations performed with well-established ray and PE propagation codes. The “microlocal” technique proposed in (Benamou et al., 2004) will be used to extract travel times of acoustic multipaths from the full-field calculations.

The key individuals that have been involved in this work are Oleg A. Godin (CIRES/Univ. of Colorado and NOAA/ESRL) and Nikolay A. Zabotin (CIRES/Univ. of Colorado). Dr. Zabotin focused on developing and testing an efficient computer code for modeling acoustic wavefront propagation in an ocean with range-dependent sound speed and current velocity. Dr. Godin took the lead in theoretical description of effects of ocean currents and localized inhomogeneities on acoustic wavefronts.

## WORK COMPLETED

Distortions of wavefronts by localized inhomogeneities have been studied by deriving elementary asymptotic solutions in the frequency and time domains for the acoustic Green's functions in a homogeneous fluid with spherical inclusions (Godin, 2011a, 2011c).

An exact wave equation has been derived for sound in inhomogeneous, moving, and non-stationary fluids under rather general assumptions about dependence of the sound speed, mass density, and ambient flow velocity on position and time (Godin, 2011d). The exact wave equation has been used to extend the validity conditions of certain previously derived approximate acoustic wave equations for moving fluids and to gain insights into the effects of the internal gravity wave-induced currents on sound propagation.

Formation of regular, deterministic wavefronts as a result of interference of sound waves scattered by rough interfaces has been investigated for the case of a shallow-water waveguide (Godin, 2011b).

Based on an open source wavefront tracing code, originally created for seismic applications (Sava and Fomel, 2001), software has been developed to simulate acoustic wavefronts and timefronts in the ocean modeled as a moving fluid with range-dependent sound speed and current velocity. The acoustic wavefront tracing code has been benchmarked using analytic solutions of the eikonal equation in moving and motionless fluids.

## RESULTS

A software package has been developed for numerical modeling of acoustic wavefronts and timefronts in range-dependent ocean. We model wavefront propagation without solving ray equations. (Rays are recovered, though, as a by-product of the wavefront construction.) The new software is an adaptation and extension of the computer codes originally developed by Sava and Fomel (2001) for seismic modeling and imaging. These codes implement an approach known as Huygens wavefront tracing (HWT) (Sava and Fomel, 2001). HWT consists in solving by a finite-difference technique of a certain system of partial differential equations, which is equivalent to the eikonal equation but is formulated in the ray coordinate system. This should be contrasted with traditional ray tracing, where the eikonal equation is solved by numerically integrating a large number of ordinary differential equations describing individual rays. For wavefront tracing in inhomogeneous media, HWT is much more computationally efficient and robust than traditional ray codes (Sava and Fomel, 2001). Unlike many other eikonal solvers, the HWT method produces the output in ray coordinates and has the important ability to track multiple arrivals. With the HWT method, each wavefront is generated from the preceding one by finite differences in the ray-coordinates domain. The first-order discretization scheme for HWT has a remarkably simple interpretation in terms of the Huygens principle, hence the method's name. Basic algorithm implementing HWT is a part of an open source *Madagascar* project, see [http://www.reproducibility.org/wiki/Main\\_Page](http://www.reproducibility.org/wiki/Main_Page). The choice of HWT as the basis of acoustic wavefront modeling in the ocean was also motivated by the fact that the Huygens principle remains valid in moving media (Godin, 1998) and, therefore, it should be possible to describe the acoustic effects of oceanic currents within the HWT framework.

As a part of adaptation of the HWT technique to underwater acoustics problem, it proved necessary to modify the underlying finite-difference algorithm to improve stability of wavefront predictions at long-range propagation in media with a wide range of spatial scales of inhomogeneities. Using theoretical results on properties of acoustic rays and wavefronts in inhomogeneous anisotropic environments (Godin, 2009), we have extended the HWT technique to moving fluids. Functionality has been developed to efficiently model acoustic timefronts in addition to the wavefronts.

To verify the new numerical model of wavefront propagation, we have formulated several benchmark problems. Acoustic wavefronts due to a stationary point source in a homogeneous fluid uniformly moving with a speed, which is less than the sound speed, are non-concentric spheres. Comparison of the exact analytic solution (Brekhovskikh and Godin, 1999) with numerical predictions is illustrated in Figure 1 for four wavefronts, which correspond to travel times  $t = 2, 4, 6$ , and  $8$  s in a fluid with the sound speed  $c = 1480$  m/s and the flow velocity  $\mathbf{u} = (500 \text{ m/s}, 0, 0)$ .

New explicit, analytic solutions of the eikonal equation have been found for wavefronts due to a point source in fluids with a uniform flow and linear profiles of either the sound speed or the refractive index squared, as well as for a fluid with linear profiles of the sound speed and the flow velocity. Figure 2 shows a comparison of the analytic and numerical solutions for acoustic timefronts due a stationary point source in a fluid with sound speed  $c = \alpha z$  and flow velocity  $\mathbf{u} = (u_0(1 + \gamma z), 0, 0)$ , where  $\alpha = 0.1 \text{ s}^{-1}$ ,  $u_0 = 200 \text{ m/s}$ , and  $\gamma = 5 \cdot 10^{-5} \text{ m}^{-1}$ . Exaggerated values of flow velocities have been utilized to illustrate accuracy of numerical predictions under more stringent conditions.

Application of wavefront tracing to simulation of long-range sound propagation in the ocean is illustrated in Figure 3, where timefronts in range-independent ocean are compared to timefronts in a single realization of an ocean with a random internal gravity wave field described by the Garrett-Munk spectrum. The high-resolution, multi-mode model of the internal gravity wave field employed in our simulations is described in (Godin et al., 2006). At the present stage, the internal wave model predicts only random sound-speed perturbations but not the random currents caused by the internal waves; three-dimensional effects are not yet accounted for in the sound propagation model.

Within the geometric acoustics approximation, the new wavefront-tracing software allows one to efficiently model effects of “spice,” internal gravity waves, and other physical processes in the water column on temporal structure of the acoustic field and on the spatial structure of wavefronts and timefronts. Acoustic travel-time bias, multi-pathing, rapid proliferation of eigenrays, penetration into shadow zones, and other previously studied effects caused by random internal gravity waves (Brown et al., 2003; Godin et al., 2006; Virovlyansky, 2006; Godin, 2007; Makarov et al., 2010) are clearly seen in Figure 3. Computational efficiency of the wavefront-tracing algorithms makes the approach suitable for large-scale Monte-Carlo simulations.

## IMPACT/APPLICATIONS

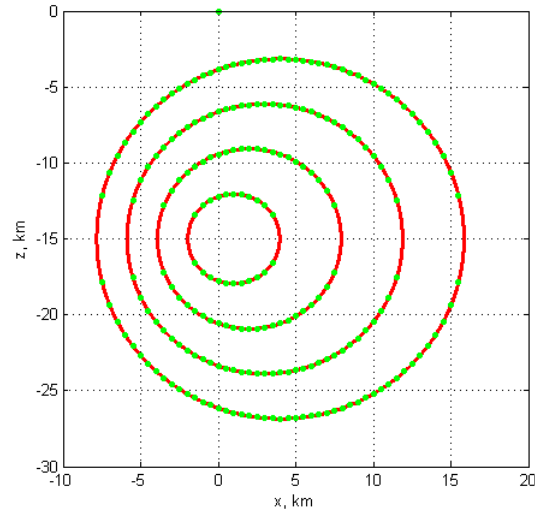
Our preliminary results indicate that numerical modeling of propagation of acoustic wavefronts in the ocean can be an effective and robust alternative to ray tracing.

## RELATED PROJECTS

None.

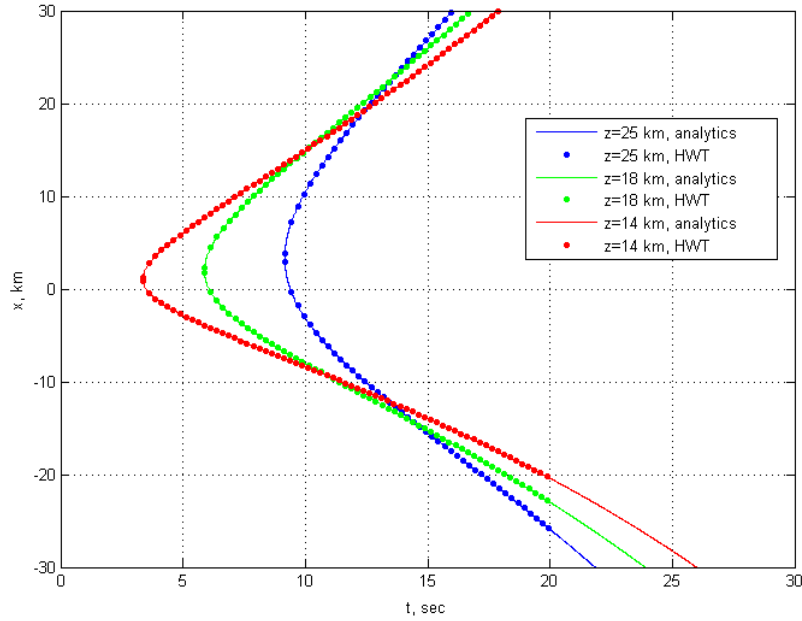
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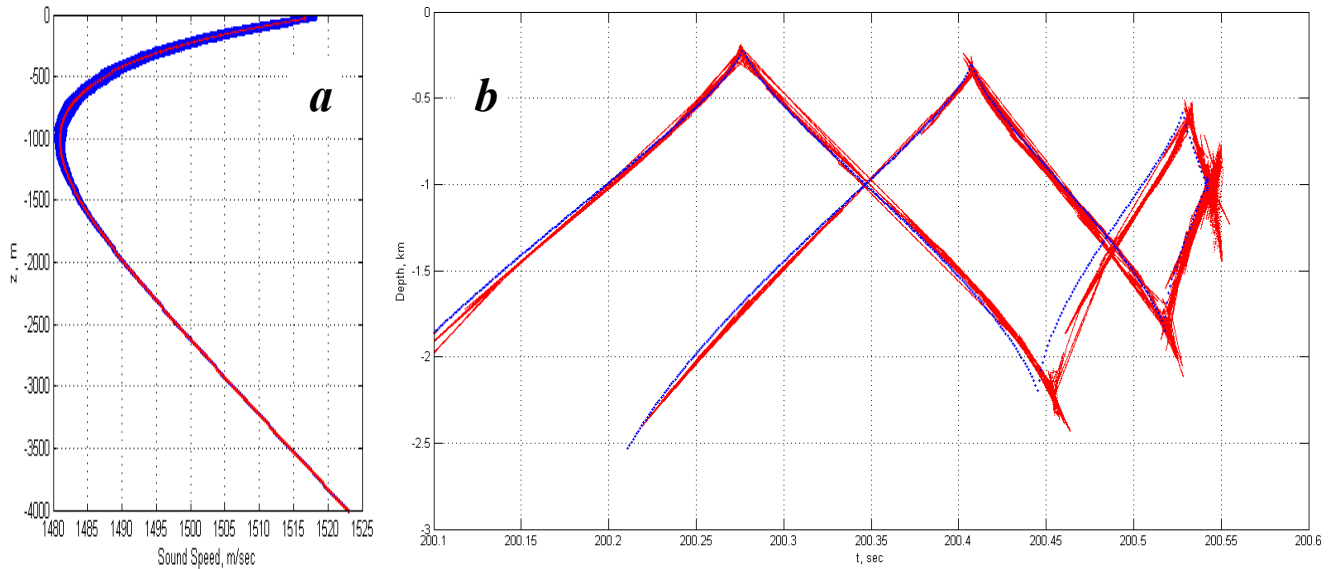
***Figure 1. Acoustic wavefronts in a homogeneous, uniformly moving fluid. Sound source is located at the point with coordinates  $(0, 0, -15 \text{ km})$ . Analytic and numerical solutions are shown in green and red, respectively.***

***[Shown in the  $xz$  plane, wavefronts are circles with their radii increasing with time and their centers moving in the direction of the ambient flow. Analytic and numerical results are in an excellent agreement.]***



**Figure 2. Acoustic timefronts in a fluid with linear sound speed and current velocity profiles. The sound speed and current velocity are functions of depth  $z$ . Current velocity is parallel to  $Ox$  coordinate axis. Sound source is located at the point with coordinates  $(0, 0, 10 \text{ km})$ . Timefronts are shown in the  $xz$  plane for  $z = 14, 18$ , and  $25 \text{ km}$ . Analytic (lines) and numerical (dots) solutions are in an excellent agreement.**

**[The timefronts are shown for the horizontal coordinate  $x$  between  $-30$  and  $30 \text{ km}$  and acoustic travel time  $t$  between  $4$  and  $26 \text{ sec}$ . Because of the currents, the timefronts are not symmetrical with respect to the  $x = 0$  line.]**



**Figure 3. Acoustic timefronts on a vertical line array at the range 297 km in an ocean with internal gravity waves. A point source is located at depth 1 km, which is close to the axis of the underwater sound channel. In panel (a), sound speed profiles at different ranges and range-averaged sound speed profile are shown in blue and red, respectively. In panel (b), timefronts in the presence of internal gravity waves (red) are compared to the timefronts (blue) in a range-independent ocean.**

**[In panel (a), sound speed varies between about 1480 and 1524 m/s. Sound speed profiles have a minimum around 1 km depth. Internal wave-induced variations in the sound speed occur primarily above 1.2 km depth. In panel (b), timefronts are shown for travel times between 200.1 s and 200.6 s and depths between about 0.2 and 2.6 km. Internal waves effectively shift acoustic timefronts in time, extend in depth, and replace each branch of a timefront by a number of near-parallel branches.]**

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